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COMPUTATION OF LAMINAR VISCOUS FLUID FLOWS IN ARBITRARY AXISYMMETRIC CHANNELS

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A finite-difference method is proposed for computing flows in axisymmetric channels of arbitrary configuration in the presence of a swirling stream.

One of the widespread causes of fluid flows in modern power plant elements is axisymmetric motion. It is characteristic for diffuser and expander type channels, axiradial turbine channels, different kinds of branchpieces and is accompanied sufficiently often by a swirling stream that raises the intensity of the heat and mass transfer processes that occur.

Laminar fluid flow in the initial section of a straight annular channel is studied in the presence of a swirling stream in [1], while an annular channel with arbitrary generators is examined in [2]. The formation of stream separation zones near the channel walls has been established.

Swirling fluid flows in a straight cylindrical pipe without a central body have been examined in [3, 4]. A reversible flow domain with several recirculation centers occurs on the pipe axis for high values of the rotation parameter.

A computation of axiradial channels of arbitrary configuration in the presence of a swirling stream is performed in [5]. The influence of the Reynolds number and the rotation parameter on the fundamental stream characteristics has been investigated. An analogous problem is solved in [6] without taking swirling into account.

The stream function, vorticity, and the circumferential velocity are the main dependent variables in [1-6]. However, solution of the Navier—Stokes equations is realized more and more often with respect to the so-called physical variables (the velocity and pressure components). The mode of writing the Navier—Stokes equations that characterize fluid flow in arbitrary axisymmetric channels is set down below and a difference method is proposed for the solution of such problems.

As is known [8], the nonstationary motion of an incompressible viscous fluid in an arbitrary curvilinear nonorthogonal coordinate system x^1, x^2, x^3 is described by the following equations

$$\frac{\partial v_i}{\partial t}+\frac{\partial(\hat{v}^k\hat{v}_i)}{\partial x^k}=-\frac{\partial p}{\partial x^i}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x^k}\left(\hat{g}^{kl}\frac{\partial\hat{v}_i}{\partial x^l}\right), \tag{1}$$

$$\frac{\partial\hat{v}^k}{\partial x^k}=0,\ i,\ k,\ l=1,\ 2,\ 3. \tag{2}$$

Here and henceforth, subscripts repeated twice assume summation over all their allowable values. The velocity vector components in the x^1, x^2, x^3 coordinate system are related to the Cartesian components by known tensory analysis relationships (α = 1, 2, 3):

$$v_i=u_\alpha\frac{\partial y_\alpha}{\partial x^i},\quad v^i=u_\alpha\frac{\partial x^i}{\partial y_\alpha},\quad u_\alpha=v_i\frac{\partial x^i}{\partial y_\alpha}=v^i\frac{\partial y_\alpha}{\partial x^i}, \tag{3}$$

while the quantities $\hat{v}_i$, $\hat{v}^i$, $\hat{g}^{k\ell}$ are determined by using matrices of the derivatives $\partial x^i/\partial y_\alpha$ and $\partial y_\alpha/\partial x^i$ fixed at the point of differentiation Q:

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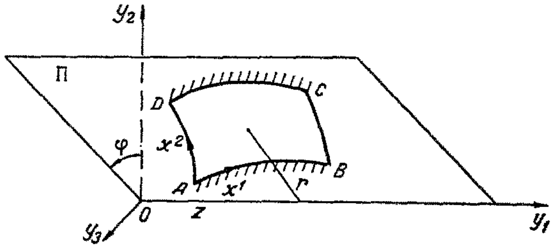

Fig. 1. Meridian section of an axisymmetric channel

$$\hat{v}_i = u_\alpha \left(\frac{\partial y_\alpha}{\partial x^i}\right)_Q, \quad \hat{v}^i = u_\alpha \left(\frac{\partial x^i}{\partial y_\alpha}\right)_Q, \quad \hat{g}^{kl} = \frac{\partial x^l}{\partial y_\alpha}\left(\frac{\partial x^k}{\partial y_\alpha}\right)_Q. \tag{4}$$

Let us now consider the case of axisymmetric fluid motion in a channel in the presence of a swirling stream. Let the Oy_1 axis be the channel axis of symmetry (Fig. 1). Let us introduce the meridian plane Π making the arbitrary angle φ with the plane Oy_1y_2. Let us assume that the meridian section of an axisymmetric channel is a quadrangular figure ABCD with arbitrary curvilinear boundaries. Let us select a curvilinear system (x^1, x^2), nonorthogonal in the general case, in the plane Π whose coordinate lines agree with the boundaries of the domain under investigation. The quadrangular figure ABCD is converted into a canonical figure in this system, the rectangle $(0 \le x^1 \le a,\ 0 \le x^2 \le b)$. We select the angle of rotation φ of the plane Π around the axis Oy_1 as coordinate x^3, i.e., φ.

The Cartesian coordinates of any point of the plane Π are determined by the relationships

$$y_1 = z, \ y_2 = r\cos x^3, \ y_3 = r\sin x^3, \tag{5}$$

where $r = r(x^1, x^2)$, $z = z(x^1, x^2)$ for an axisymmetric channel. It follows from (5) that for $x^3 = 0$

$$\frac{\partial y_1}{\partial x^1} = \frac{\partial z}{\partial x^1}, \quad \frac{\partial y_1}{\partial x^2} = \frac{\partial z}{\partial x^2}, \quad \frac{\partial y_2}{\partial x^1} = \frac{\partial r}{\partial x^1}, \quad \frac{\partial y_2}{\partial x^2} = \frac{\partial r}{\partial x^2},$$
$$\frac{\partial y_1}{\partial x^3} = \frac{\partial y_2}{\partial x^3} = \frac{\partial y_3}{\partial x^1} = \frac{\partial y_3}{\partial x^2} = 0, \quad \frac{\partial y_3}{\partial x^3} = r. \tag{6}$$

Assuming a coordinate transformation inverse to (5) to exist, we can obtain for $x^3 = 0$:

$$\frac{\partial x^1}{\partial y_1} = \frac{1}{g}\frac{\partial r}{\partial x^2}, \quad \frac{\partial x^1}{\partial y_2} = -\frac{1}{g}\frac{\partial z}{\partial x^2},$$
$$\frac{\partial x^2}{\partial y_1} = -\frac{1}{g}\frac{\partial r}{\partial x^1}, \quad \frac{\partial x^2}{\partial y_2} = \frac{1}{g}\frac{\partial z}{\partial x^1}, \tag{7}$$
$$\frac{\partial x^1}{\partial y_3} = \frac{\partial x^2}{\partial y_3} = \frac{\partial x^3}{\partial y_1} = \frac{\partial x^3}{\partial y_2} = 0, \quad \frac{\partial x^3}{\partial y_3} = \frac{1}{r},$$

and also

$$\frac{\partial}{\partial x^3}\left(\frac{\partial x^1}{\partial y_3}\right) = \frac{\partial x^1}{\partial y_2}, \quad \frac{\partial}{\partial x^3}\left(\frac{\partial x^2}{\partial y_3}\right) = \frac{\partial x^2}{\partial y_2}, \quad \frac{\partial}{\partial x^3}\left(\frac{\partial x^3}{\partial y_3}\right) = 0, \tag{8}$$

where

$$g = \frac{\partial z}{\partial x^1}\frac{\partial r}{\partial x^2} - \frac{\partial z}{\partial x^2}\frac{\partial r}{\partial x^1}.$$

Now, let us consider the kinematic characteristics of the axisymmetric motion. Let U, V, W be axial, radial, and circumferential velocities, independent of x^3, in the cylindrical coordinate system. Then the dependence of the Cartesian velocity components on the coordinate x^3 is determined by the formulas

$$u_1 = U, \ u_2 = V\cos x^3 - W\sin x^3, \ u_3 = V\sin x^3 + W\cos x^3. \tag{9}$$

Using (3), (6), (7) and (9), we obtain expressions for the functions $\hat{v}_i$ and $\hat{v}^i$:

$$\hat{v}_1 = U \frac{\partial z}{\partial x^1} + (V \cos x^3 - W \sin x^3) \frac{\partial r}{\partial x^1},$$
$$\hat{v}_2 = U \frac{\partial z}{\partial x^2} + (V \cos x^3 - W \sin x^3) \frac{\partial r}{\partial x^2}, \qquad (10)$$
$$\hat{v}_3 = (V \sin x^3 + W \cos x^3)\, r;$$

$$\hat{v}^1 = \frac{1}{g}\left[U \frac{\partial r}{\partial x^2} - (V \cos x^3 - W \sin x^3) \frac{\partial z}{\partial x^2}\right],$$
$$\hat{v}^2 = \frac{1}{g}\left[-U \frac{\partial r}{\partial x^1} + (V \cos x^3 - W \sin x^3) \frac{\partial z}{\partial x^1}\right], \qquad (11)$$
$$\hat{v}^3 = \frac{1}{r}(V \sin x^3 + W \cos x^3).$$

The quantities r, g, $\partial r/\partial x^1$, $\partial r/\partial x^2$, $\partial z/\partial x^1$ and $\partial z/\partial x^2$ in (10) and (11) should be considered fixed at the point of differentiation Q located in the plane $x^3 = 0$.

As follows from (10) and (11), the values of $\hat{v}_i$ and $\hat{v}^i$ depend on the angular coordinate x^3. consequently, to obtain the Navier–Stokes equations describing the axisymmetric motion case, it is necessary to expand analytically the derivatives with respect to the coordinate x^3 that enter into (1) and (2). By virtue of axial symmetry it is sufficient to execute this procedure in some one meridian plane, the plane $x^3 = 0$, say.

Let us first examine the expression in the left side of the continuity equation (2) and let us extract the derivative with respect to the coordinate x^3

$$\frac{\partial \hat{v}^k}{\partial x^k} = \frac{\partial \hat{v}^s}{\partial x^s} + \frac{\partial \hat{v}^3}{\partial x^3}, \quad k = 1, 2, 3; \quad s = 1, 2.$$

Taking account of (3), (6), and (11), we will have for $x^3 = 0$

$$\frac{\partial \hat{v}^3}{\partial x^3} = \frac{V}{r} = \frac{v^s}{r} \frac{\partial r}{\partial x^s}, \quad s = 1, 2. \qquad (12)$$

In this case

$$\frac{\partial \hat{v}^k}{\partial x^k} = \frac{1}{r} \frac{\partial}{\partial x^s}(r\hat{v}^s), \quad k = 1, 2, 3; \; s = 1, 2. \qquad (13)$$

Now, let us consider the convective terms of the momentum equations (1)

$$\frac{\partial (\hat{v}^k \hat{v}_i)}{\partial x^k} = \frac{\partial (\hat{v}^s \hat{v}_i)}{\partial x^s} + v_i \frac{\partial \hat{v}^3}{\partial x^3} + v^3 \frac{\partial \hat{v}_i}{\partial x^3}, \quad k = 1, 2, 3; \; s = 1, 2.$$

Taking (10)-(12) into account into account, we will have for $x^3 = 0$ (s = 1, 2)

$$v_i \frac{\partial \hat{v}^3}{\partial x^3} = \frac{1}{r} v_i v^s \frac{\partial r}{\partial x^s}, \quad v^3 \frac{\partial \hat{v}_i}{\partial x^3} = K_i,$$

where

$$K_1 = -\frac{W^2}{r} \frac{\partial r}{\partial x^1}; \quad K_2 = -\frac{W^2}{r} \frac{\partial r}{\partial x^2}; \quad K_3 = VW. \qquad (14)$$

In this case

$$\frac{\partial (\hat{v}^k \hat{v}_i)}{\partial x^k} = \frac{1}{r} \frac{\partial}{\partial x^s}(r\hat{v}^s \hat{v}_i) + K_i, \quad k = 1, 2, 3; \; s = 1, 2. \qquad (15)$$

Finally, the diffusion terms of the Navier–Stokes equations can be reduced to the following form by using (6)-(8):

$$\frac{\partial}{\partial x^k}\left(\hat{g}^{kl} \frac{\partial \hat{v}_i}{\partial x^l}\right) = \frac{1}{r} \frac{\partial}{\partial x^s}\left(r\hat{g}^{sm} \frac{\partial \hat{v}_i}{\partial x^m}\right) + D_i, \qquad (16)$$
$$k, l = 1, 2, 3; \; s, m = 1, 2,$$

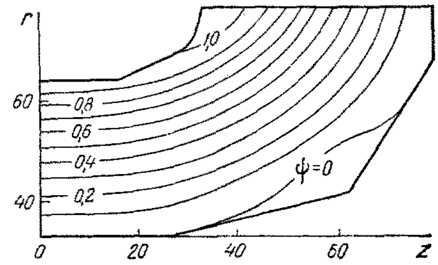

Fig. 2. Stream lines in an axiradial diffuser, Re = 1000.

here

$$D_1 = -\frac{V}{r^2}\frac{\partial r}{\partial x^1},\ D_2 = -\frac{V}{r^2}\frac{\partial r}{\partial x^2},\ D_3 = -\frac{W}{r}. \tag{17}$$

Using the expressions (13)-(17) obtained, we convert the system of Navier–Stokes equations (1) and (2) to universal form describing the motion of an incompressible viscous fluid in both the plane and axisymmetric cases in the presence of stream swirling

$$\frac{\partial v_i}{\partial t} + \frac{1}{r^\sigma}\frac{\partial}{\partial x^s}(r^\sigma \hat{v}^s \hat{v}_i) = -\frac{\partial p}{\partial x^i} + \frac{1}{\mathrm{Re}\, r^\sigma}\frac{\partial}{\partial x^s}\left(r^\sigma \hat{g}^{sm}\frac{\partial \hat{v}_i}{\partial x^m}\right) + \sigma I_i, \tag{18}$$

$$\frac{1}{r^\sigma}\frac{\partial}{\partial x^s}(r^\sigma \hat{v}^s) = 0,\ s,\ m = 1,\ 2, \tag{19}$$

where

$$I_1 = \frac{1}{r}\left(W^2 - \frac{V}{\mathrm{Re}\, r}\right)\frac{\partial r}{\partial x^1},$$

$$I_2 = \frac{1}{r}\left(W^2 - \frac{V}{\mathrm{Re}\, r}\right)\frac{\partial r}{\partial x^2},\ I_3 = -\frac{W}{r}\left(rV + \frac{1}{\mathrm{Re}}\right).$$

When considering plane channels, we should set $\sigma = 0$, $i = 1, 2$ in (18) and (19). In the case of an axisymmetric flow with a swirling stream $\sigma = 1$, $i = 1, 2, 3$ and $\partial p/\partial x^3 = 0$.

The system of equations (18) and (19) is closed by the following boundary conditions. All velocity vector components $v_1 = v_{10}(x^2)$, $v_2 = v_{20}(x^2)$, $v_3 = v_{30}(x^2)$ are given at the entrance to the channel ($x^1 = 0$). The usual conditions of adhesion and non-penetration $v_1 = v_2 = v_3 = 0$ are posed on the solid boundaries ($x^2 = 0$ and $x^2 = b$). Finally, the boundary conditions at the exit from the channel ($x^1 = a$) are written in the form [8]

$$\frac{\partial}{\partial x^1}\frac{r^\sigma}{g}\left\{v_1\left[\left(\frac{\partial r}{\partial x^2}\right)^2 + \left(\frac{\partial z}{\partial x^2}\right)^2\right] - v_2\left(\frac{\partial r}{\partial x^1}\frac{\partial r}{\partial x^2} + \frac{\partial z}{\partial x^1}\frac{\partial z}{\partial x^2}\right)\right\} = 0,\quad \frac{\partial v_2}{\partial x^1} = \frac{\partial v_3}{\partial x^1} = 0. \tag{20}$$

The first condition in (20) denotes constancy of the fluid mass flow rate through the side faces $dx^2 \times dx^3$ of the volume element $dx^1 \times dx^2 \times dx^3$ adjoining the channel output section. These conditions in a Cartesian coordinate system go over into the usual "soft" conditions. As follows from (18), when solving the problem in physical variables the pressure p is determined to the accuracy of an arbitrary constant. This constant is found from the condition $p = 0$ at the point $x^1 = x^2 = 0$.

The procedure of generating a difference mesh to be used to find the relation between coordinates of points of the physical and canonical domains, i.e., $r = r(x^1, x^2)$, $z = z(x^1, x^2)$, preceded the direct numerical modeling of viscous fluid motion on the basis of (18) and (19). The algorithm for construction of the mesh is based on using a generating system of elliptical equations [9]. Upon placement of the nodes along the solid boundaries, their condensation in domains of positive curvature of the contour is assured, where the appearance of stream separations is most probable. The functions regulating the transverse condensation of the nodes are calculated with the exponential behavior taken into account near the walls. The procedure assures construction of a mesh sufficiently close to an orthogonal one.

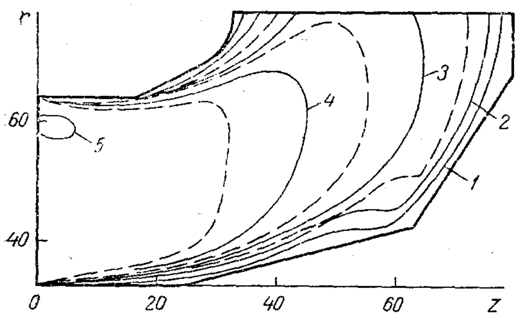

Fig. 3. Circumferential velocity isolines in an axiradial diffusor: 1) W = 0.1, 2) 0.2, 3) 0.3, 4) 0.4, 5) 0.5, Re = 1000.

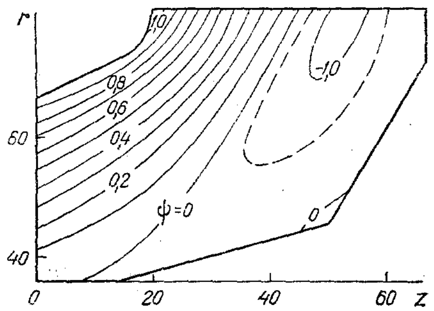

Fig. 4. Stream lines in a truncated axiradial diffuser with nonuniform velocity profile at the entrance and a swirling stream, Re = 1000.

We will determine the desired mesh functions p and v_3 at the center of each mesh cell and the functions v_1 and v_2 at the center of its faces exactly as is customary in the method of markers and cells. We denote the approximation of the derivative ∂/∂_{x^s} by central differences by D_s and the approximation of the convective terms $\partial(r^\sigma\hat{v}^s\hat{v}_i)/\partial x^s$ in (18) by the scheme of donor cells by $D_s^*(r^\sigma\hat{v}^s, \hat{v}_i)$.

We use the following multistep implicit difference scheme to solve the system of equations (18) and (19) (the superscript n is the number of the time layer)

$$(v_3^{n+1}-v_3^n)/\Delta t + r^{-\sigma}D_s^*(r^\sigma\hat{v}^{sn}, \hat{v}_3^{n+1}) = (\mathrm{Re}\, r^\sigma)^{-1}D_s(r^\sigma\hat{g}^{sm}D_m(\hat{v}_3^{n+1})) + \sigma I_3^{n+1}, \tag{21}$$

$$\delta v_j^{n+1/3}/\Delta t + r^{-\sigma}D_s^*(r^\sigma\hat{v}^{sn}, \hat{v}_j^n) = -D_j(p^n) + (\mathrm{Re}\, r^\sigma)^{-1}D_s(r^\sigma\hat{g}^{sm}D_m(\hat{v}_j^n)) + \sigma I_j^{n+1}, \tag{22}$$

$$D_s(r^\sigma\hat{g}^{sj}(v_j^n + \delta v_j^{n+1/3} - \Delta t D_j(\delta p))) = 0, \tag{23}$$

$$(\delta v_j^{n+2/3} - \delta v_j^{n+1/3})/\Delta t = -D_j(\delta p), \tag{24}$$

$$(\delta v_j^{n+1} - \delta v_j^{n+2/3})/\Delta t + r^{-\sigma}D_s^*(r^\sigma\hat{v}^{sn}, \delta v_j^{n+1}) = (\mathrm{Re}\, r^\sigma)^{-1}D_s(r^\sigma\hat{g}^{sm}D_m(\delta v_j^{n+1})), \tag{25}$$

$$v_j^{n+1} = v_j^n + \delta v_j^{n+1}, \quad p^{n+1} = p^n + \delta p, \quad j = 1, 2. \tag{26}$$

The scheme (21)-(26) is an extension of the difference scheme proposed in [8] to compute plane flows in arbitrary channels, to the axisymmetric swirling fluid flow case. Its main steps are the following.

By using (21) values of the functions v_3^{n+1} at the (n + 1)-th time layer are determined by iteration from values given for the mesh functions v_1^n, v_2^n, v_3^n and p^n at the n-th time layer. Then the preliminary corrections $\delta v_j^{n+1/3}$ (j = 1, 2) to the velocities are found from the explicit formulas (22). Furthermore, the field of corrections to the pressure δp is determined from (23) by iterations. New corrections to the velocities $\delta v_j^{n+2/3}$ are determined from (24) by a simple conversion, and the final corrections δv_j^{n+1} are found by iteration from (25). Finally, values of the velocities v_j^{n+1} (j = 1, 2) and pressure p^{n+1} are calculated at the new (n + 1)-th time layer by using (26). The process is repeated from the very beginning to obtain solutions steady-state in time.

The implicit steps (21), (23), and (25) of the proposed algorithm are realized by using the splitting method. A sequence of steps in the relaxation time is used to accelerate the iteration process in the solution of (23). Splitting (21) and (25) is determined by the stream direction which permits especially effective investigation of a flow with recirculation zones [8]. The difference scheme used in this paper satisfies the test of a homogeneous stream, which is necessary in computing fluid flows in a curvilinear nonorthogonal coordinate system [9].

The finite-difference method described above underlies the program complex developed for computation of laminar incompressible viscous fluid flows in plane and axisymmetric channels of arbitrary configuration. Stream swirling is taken into account in the axisymmetric case. The flow domain boundaries can be given by oints, arcs of circles, or segments of straight lines. Breakpoints in the body surface, baffles, stages, etc. are allowed here.

Computations of swirling flows in axisradial diffusors of different configuration that are applied in modern turbines were performed by using the program complex. The stream Reynolds number, and the shape of the velocity component profile at the entrance to the channel were varied within a broad range. The program complex yields similar flow patterns for each of the computation modifications: stream lines, velocity and swirling velocity isolines, etc. The integral flow characteristics are also calculated.

Streamlines in an axiradial diffuser whose boundaries are formed by arc and straight line segments are represented in Fig. 2. Homogeneous longitudinal (U_0 = 1) and circumferential (W_0 = 0.5) velocity profiles are given at the channel entrance, there is not radial velocity. The Reynolds number constructed along the entrance width is 1000. As follows from the figure, a closed recirculation domain is formed at the channel inner surface (fairing) near the boundary breakpoint. Stream separation is observed at the outer surface of the diffusor near its exit. The circumferential velocity isolines corresponding to the motion case under consideration are presented in Fig. 3 with the step $\Delta W = 0.1$. The intermediate dashed isolines are superposed with the step $\Delta W = 0.05$. It can be noted that as the longitudinal coordinate increases dissipation of the swirling occurs. The circumferential velocity isolines are located close to the radial lines in the central part of the channel.

Finally, let us consider an example of a fluid flow computation in a channel whose configuration is not in agreement with the boundary conditions at the entrance. Streamlines in a truncated axiradial diffusor at whose entrance velocity component profiles that are almost real are given that correspond to exit conditions from the last stage of a turbine are superposed in Fig. 4. The Reynolds number also equals 1000. As follows from the figure, powerful stream separation emerging beyond the boundaries of the computational domain develops on the fairing surface. This circumstance significantly reduces the channel efficiency and indicates the necessity to profile it according to real velocity distribution at the entrance. The program complex described permits construction of channels optimal in its characteristics by the performance of several modifications of the computation.

NOTATION

y_1, y_2, y_3, Cartesian coordinates; z, r, φ, are cylindrical coordinates; x^1, x^2, x^3, curvilinear coordinates; u_1, u_2, u_3, Cartesian velocity components; v_1, v_2, v_3, v^1, v^2, v^3, are co- and contravariant velocity components; U, V, W, axial, radial, and circumferential velocities; $g^{k\ell}$, metric tensor components; t, time; p, pressure; Re, Reynolds number; Δt, time step; δ, function increment; and ψ, stream function.

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SUPER-MOLECULAR STRUCTURE OF DILUTE SOLUTIONS OF HIGH MOLECULAR WEIGHT POLYMERS WHICH LEAD TO REDUCED TURBULENT FRICTION

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Data are obtained which indicate a relationship between the viscoelastic properties of dilute solutions of high molecular weight polymers and their super-molecular network structures. The changes in structure are determined which result as the concentration and molecular weight of the polymer are increased.

Dilute solutions of linear high molecular weight polymers with their unusual hydrodynamic and physical-chemical properties have long attracted the attention of investigators. The reduction of turbulent friction, increase of the resistance to filtration, the suppression of the breakup of jets, and the flocculating effect of extremely small polymer additions are all effects which have not only theoretical interest but also considerable applied value. The progress in understanding the special features of dilute polymer solutions is closely related to the development of modern concepts of the structural features of these liquids. However, it is precisely on this basic question that no unified opinion has been developed up to now.

A point of view is widely encountered in polymer science according to which the polymer chains in dilute solutions are molecularly dispersed. In this sense a dilute solution is taken to mean a solution in which there is no overlapping of the macromolecular tangles, which in the case when the limit exists $[\eta]_0 = \lim_{c \to 0,\, \dot{\gamma} \to 0} (\eta - \eta_s)/(c\eta_s)$, (which is termed the characteristic viscosity) results in the condition $c[\eta]_0 < 1$ being satisfied. Consistent experimental information exists in favor of the molecular dispersity of dilute solutions for polymeric materials with molecular weights $M \leqslant 10^5$. In this connection it is sufficient to recall the classical results of H. Staudinger and W. Carothers and other authors, who determined the molecular weights of polymers by basically different methods: chemical (titration of the end groups), and physical (cryoscopy, ebullioscopy, osmometry) [1]. These experiments led to comparable data, and indicated not only the existence of polymers with molecular weights exceeding 10^4, but also the molecular dispersity of these materials in solution. It was possible to advance these results by still another order of magnitude with respect to M after the development of the light-scattering method by Debye. The use of light scattering and osmometry for measuring the molecular weights of polymers reaching values as high as 10^5 led to the same numerical values, which inidcates, in particular, the correctness of the concepts of the molecular state of subdivision of such polymers in dilute solutions [2].

The successes in proving the separateness and discreteness of macromolecules with $M \leqslant 10^5$ in dilute solutions led to the unfounded confidence that this discreteness is retained for dilute solutions of macromolecules with larger molecular weights also, i.e., for the solutions for which the series of effects listed above are characteristic. In particular, this confidence is related to the attempts which have been made to explain many of the hydrodynamic features of the behavior of liquids with small polymeric additions on the basis of considering the interactions of single macromolecules with the flows. The point of view being discussed here has become particularly popular among theoreticians as a result of the tempting possibilities it provides for simplifying calculations.

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